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ABSTRACT

A computer-assisted instructional package designed to teach both specific series-solving skills and general problem-solving skills was run in a school environment for one year. Students had access to the program during their free time. This paper analyzes the progress made by one student in both series solving and general problem solving during his four hours of work on the program. (Author)

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THE SERIES PROGRAM: ONE STUDENT'S BEHAVIOR

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Abstract

A computer-assisted instructional package designed to teach both specific series-solving skills and general problem-solving skills was run in a school environment for one year. Students had access to the program during their free time. This paper analyzes the progress made by one student in both series solving and general problem solving during his four hours of work on the program.

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THE SERIES PROGRAM: ONE STUDENT'S BEHAVIOR

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Several computer programs for use by elementary school students have been designed by the Computer-Assisted Instruction in Problem-Solving Project at the Learning Research and Development Center. One of these, the Series program, is designed to teach specific series-solving skills and general problem-solving skills. To assess the program's effectiveness, two studies have been undertaken. In the first, a laboratory controlled study, the learning and transfer of series-solving skills were measured (Heller, Note 1). This paper reports the second, a study of one student's entire history on the program, including both his series learning and his general problem-solving learning.

The introduction describes the program as the student saw it, the way in which the series were chosen, and the context in which the student worked. In the next section, the student's work and learning with respect to series-solving skills is described. The final section describes the same work, but focuses on general problem-solving skills.

How the Program Works

In the Series program, specific series-solving skills are learned through the selection and sequencing of the Series puzzles themselves. General problem-solving skills are learned through the environment

created for the students while they are working on the program. In this environment, students are placed in complete control of several aspects of their experience. Students select the level of difficulty for each series seen; they control the amount of time on each series by indicating when they want to change to a new series; they receive additional information (terms of the series) only upon request. After changing to a new series, students receive feedback on their previous performance. Thus, students typically work through a cycle including selecting a level of difficulty for the next series, working on the new series for as long as desired, getting feedback on their performance, and selecting a new series. This cycle repeats until the students stop for the day; it resumes when they return to the computer terminal for a subsequent session.

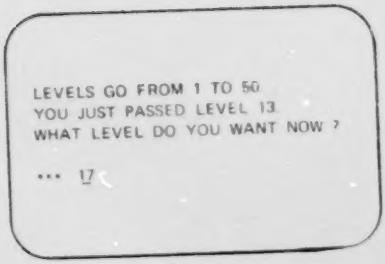
Now, consider what happens during work on each series. Once a series is selected, displayed, and work commences, the global goal for the student is to solve the series by inferring the rule of formation and generating the next terms of the series. Episodes of work involve the more immediate goal of getting the next term displayed. A new episode of behavior begins with each new term. For example, when the series "2, 5, 8, 11, 14, . . ." is displayed, the student's immediate goal is to get the next term (17) by some sequence of actions in the available repertoire. The program allows three actions: Change to a new series, abandoning the task; ask for the next term, causing the computer to display it; or guess the next term, receiving feedback on the guess. The student may attempt many guesses, but may change only once per puzzle and ask for the next term only once per term. Part of the global problem for the student is to determine when the series is understood or when the series is too difficult; these are appropriate times to change to a new series.

The displays constructed by the program convey all of the information given above in more concrete form to the students. One typical cycle

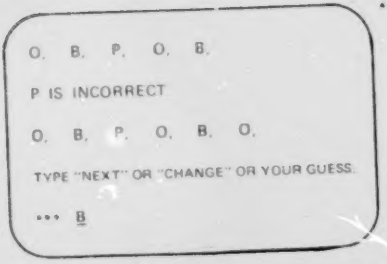
of use is illustrated in Figure 1. The initial display for a cycle is given in Figure 1, Frame 1. Notice that feedback for the previous cycle's global performance is given (YOU JUST PASSED LEVEL 13) and that the student may select from any of the 50 available levels. When the student selects a level (17 in this case), one of 40 series whose difficulty is near that level is selected at random as the next problem. The initial display for a series is illustrated in Frame 2 of Figure 1. Notice that five terms are given and that the student's options are listed. In this case, the student enters "P," a plausible, but incorrect, guess. The feedback is shown in Frame 3 of Figure 1. Notice that the student again has the same three options and is still working on the same term. The student types "NEXT," asking the computer to supply the next term. Because the student previously made an error on the term, the incorrect guess is preserved in the display so comparison is possible. Frame 4 of Figure 1 illustrates the addition of a new term by the use of "NEXT." The next two guesses were wrong ("B" and "P"), but the third guess was correct. Frame 5 of Figure 1 illustrates feedback for a correct guess after two wrong guesses. Notice the reinforcing comment, "GREAT!". The display is accompanied by the sound of a bell for correct guesses. Frame 6 illustrates the display when a correct guess follows "NEXT" or another correct guess. The previous series is erased from the screen and all the known terms are displayed at the top of the screen. As with all correct guesses, the reinforcing comment "GREAT!" is displayed and accompanied by the sound of the bell.

How Series Are Formed

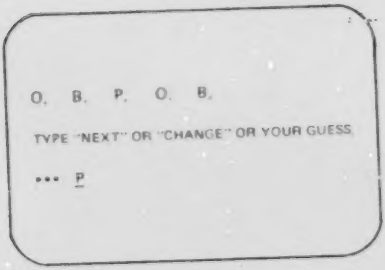
The Series program can display any of 400 general problems. Most of the 400 general problems are completed when the problem is selected by randomly choosing a starting point and/or an increment value. For example, if the general problem is to increment each term by 3, the specification might be completed by selecting a starting number between 1 and 20. Thus,



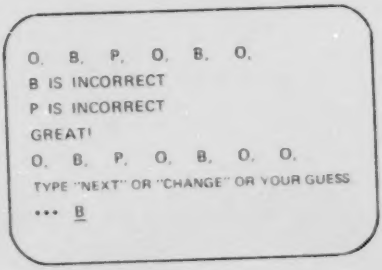
Frame 1
Initial display for cycle.



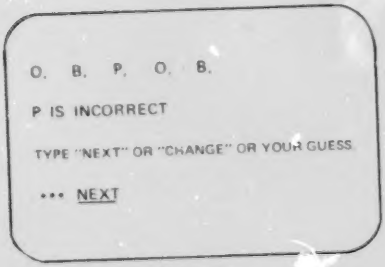
Frame 4
Display for "NEXT" after incorrect guess.



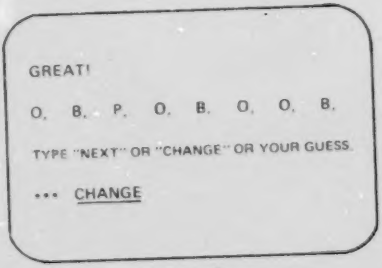
Frame 2
Initial display for series.



Frame 5
Display for correct guess
after two wrong guesses.



Frame 3
Feedback for incorrect guess.



Frame 6
Display for correct guess
after "NEXT" or correct guess.

Figure 1. A sequence of displays from the Series program.

20 specific series might be displayed from a single general problem. There are hundreds of thousands of unique series that can be displayed by the program. Although that number is large, it represents only a small fraction of all imaginable patterned sequences of letters and numbers. In this section, the principles for selecting problems and for assigning difficulty are described.

Basic series. Certain series are basic, and it is assumed that all users of the program can extend them without difficulty. These include the alphabet, the counting numbers up to 1ØØ, and any repeating pattern of one to four terms. The alphabet series can begin at any of 26 letters and continues in a cycle with "A" following "Z." The alphabet series is called "ALPHA." For a specific series, such as "R, S, T, U, V, . . . ," the name is "ALPHA starting at 'R'." The series of counting numbers is called "COUNT." The specific series "27, 28, 29, 3Ø, . . ." is named "COUNT starting at '27'." Any repeating pattern is called a "REPEAT" series. The name for the specific series "R, M, R, M, R, . . ." is "REPEAT of 'R' and 'M'." The Basic series ALPHA, COUNT, and REPEAT were assigned difficulty levels between 1 and 5. More complex series are built from the Basic series.

It is convenient to treat two other series as Basic. These are the backwards alphabet, "BALPHA," and the backwards counting numbers, "BCOUNT." These series are not recognized immediately by all students and were assigned difficulty levels between 5 and 10.

Any series can be used as a subseries in more complex series. The program uses three methods to combine simpler series into more complex series: the intersperse method, the concatenate method, and the skip method.

Intersperse series. To make a new series by the intersperse method two or more subseries are combined by taking terms alternately from each

to make the new series. Thus "R, 27, S, 28, T, 29, . . ." is an Intersperse series of an ALPHA series starting at "R" and a COUNT series starting at "27." The difficulty of an Intersperse series depends on the contrast between the terms of the subseries, the difficulty of the subseries, and the number of subseries involved in the pattern. The "contrast" is the most complex concept used in assigning difficulty levels. It can be explained best by examples. Two series derived from Basic series with the same characters have lower contrast than two derived from Basic series with different characters. Compare the difficulty of "R, 2, S, 3, T, 4, U, . . ." with "R, M, S, N, T, O, U, P, ..." Given the same characters in each subseries, contrast is higher if the terms are widely separated in the Basic series. Compare "12, 1Ø, 13, 11, 14, 12, . . ." with "12, 87, 13, 88, 14, 89, 15, 9Ø, ..." When three subseries are combined by the intersperse method contrast is low only when all three come from the same character set. Otherwise, the distinctive character set marks off the constituent subseries. Compare "1, 7, 11, 2, 8, 1Ø, 3, 9, 9, . . ." with "A, 7, 11, B, 8, 1Ø, C, 9, 9, ..." (Note: The letter "O" is distinguished from the number zero in this paper by the use of a slash across all zeroes [letter: O; number: Ø].)

Contrast is related to the difficulty because high contrast makes it easier to find the period with which the terms alternate. Kotovsky and Simon (1973) have noted the relevance of this dimension in their Find Period operator.

Concatenate series. The concatenate method for combining simpler series to make more complex series requires that terms from two or more subseries be written next to each other. The factors that make the series more difficult are analogous to those in the intersperse method. For example, the series "6M, 7L, 8K, 9J, 1Ø1, . . ." is a Concatenate series of two contrasting subseries: COUNT starting at "6" and BALPHA starting at "M." The form of the Concatenate series makes it unnecessary to find the period; each term contains a term from each subseries.

Skip series. The skip method for combining series requires a subseries and a numeric subseries that describes the skipping pattern. The series "9, 12, 15, 18, . . ." can be described as a Skip series on the COUNT series starting at "9" according to the pattern REPEAT of "3." The analogy to "D, G, J, M, P, . . ." a Skip series on the ALPHA series starting at "D" according to the pattern REPEAT of "3," makes the generality of this naming clear. The difficulty of Skip series depends on the subseries used and the complexity of the pattern. Generally, a similar pattern will be more difficult when it operates on series derived from ALPHA than on series derived from COUNT.

Combined series. Finally, the combining method can take series that are themselves combined types as subseries to create even more complex forms. For example, two subseries that are interspersed might themselves be Skip subseries. In the rest of the paper such complex series will be called "combined types."

The power of this combining method can be seen in the examples below:

ALPHA starting with "A," "A, B, C, D, E, F, . . ." (1)
and COUNT starting with "1," "1, 2, 3, 4, 5, . . ." (2)
are both assigned difficulty 1.

The Intersperse series of (1) with (2),
"A, 1, B, 2, C, 3, D, . . .", and
the Concatenate of (1) with (2), (3)
"A1, B2, C3, D4, E5, . . ." (4)
are both assigned difficulty 3.

The Skip on (1) according to pattern REPEAT of "1 and 2,"
"A, B, D, E, G, H, J, K, . . ." (5)
has difficulty 15.

Concatenate of (5) with (2),
"A1, B2, D3, E4, G5, H6, J7, . . ." (6)
has difficulty 25.

Interspersing of (6) with (4),
"A1, A1, B2, B2, C3, D3, D4, E4, E5, G5, F6, H6, . . ." (7)
has difficulty 40.

Skip on (7) according to pattern REPEAT of "5 and -2,"
"A1, D3, B2, E5, D4, H6, G5, . . ." (8)
is very likely unsolvable by grade school students.

Method

The Series program was available to students in grades 2 through 6 in an elementary school. Sixteen terminals were distributed among three learning areas in the school, and many programs were available (Block, Carlson, Fitzhugh, Hsu, Jacobson, Puente, Rosner, Simon, Glaser, & Cooley, 1973). Series was used as a selected activity on a voluntary basis after other work was completed. Usage varied from one session to over sixty sessions during the year. Typical sessions lasted ten minutes and included about seven puzzles.

Descriptions of student behavior are based upon an analysis of the printed protocols that document the student's entire Series experience. Only typed responses are represented in these protocols. Other data such as verbalizations, gestures, facial expressions, or detailed timing are not available. To infer information processes from such limited data is a difficult task. Generally, several observations are needed before inferences can be drawn and stated. In the summary that follows, excerpts from protocols will be included. Statements derived from this evidence will be explained in detail.

There are benefits to focusing on the work of only one student as we have done in this paper. Most obvious is the wealth of detail available. Each of our goals can be evaluated for this student. The particulars of behavior have led to insights about the learning process and have helped to formulate further research questions. A statistical study of all the students would mask many of the relevant details.

The student chosen for this analysis was among the 150 who used the Series program during its first operational year. At that time, any student who wished to could select the program as a free activity as frequently as he or she desired. No supervision was available while the students worked, so their performance reflects the motivational and instructional aspects of the program itself. This particular student was selected because he was among the 25 most frequent users and, therefore, data accumulated on him was sufficient to make analysis meaningful. At the time, he was 10 years, 11 months in age. His IQ score (117) was slightly above average for the school.

The full protocol consists of work on 119 series in 241 minutes. For convenience of exposition, the work has been divided into four one-hour segments, as shown in Table 1.

Table 1

Distribution of Puzzles During Student's Four Hours of Series Experience

Hour	Puzzle numbers	Number of puzzles/hour
First	1 - 34	34
Second	35 - 64	30
Third	65 - 95	31
Fourth	96 - 119	24

The next two sections of the paper discuss the development of series-solving and general problem-solving skills, respectfully. Series are divided into two categories: (a) Basic series, which include the alphabet and backward alphabet (ALPHA and BALPHA), counting (7, 8, 9, 10, . . .) both foward and backward (COUNT and BCOUNT), and REPEAT series (F, F, F, F, . . .); and (b) applications of these Basic series to combined forms of varying complexity (Skip, Intersperse, Concatenate, and combined types). The student's knowledge of both of these categories is evaluated in the section on development of series-solving skills.

Problem-solving skills in the context of Series will be described in terms of the following: (a) testing of hypotheses, (b) use of feedback from guesses, (c) gathering of additional data when needed, (d) consideration of all the data available, (e) choice of appropriate time to stop work on a series, (f) actions after groups of correct guesses, and (g) use of overall result on series for subsequent choice of new difficulty level. This discussion of problem-solving activity is contained in a later section.

Development of Series-Solving Skills

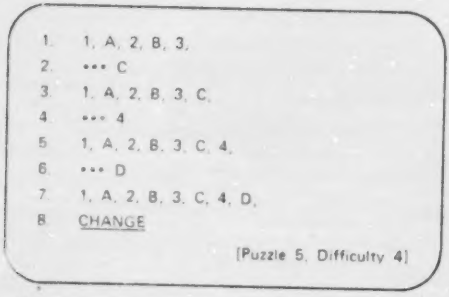
This section documents the student's learning of series-solving skills. Series are considered in the categories Basic, Concatenate, Intersperse, Skip, and combined types. The student's work in each category will be described as it changed over time.

In discussions of series-solving skills, a particular series is considered "solved" when the student gets a sufficient number of terms right in a row. "Sufficient" takes on different values depending upon the series worked on. Series with a period of one, with no describable period, or with a very lengthy period generally require two right in a row to be considered solved. Series with two or three terms in the period generally require the same number of correct responses in a row as there are terms

in a period. When the student understands part of a series, such partial solutions will also be considered and described.

Basic Series

The student made no errors in identifying and extending the Basic series ALPHA, BALPHA, COUNT, and BCOUNT during his first hour of work. He typically entered two or three correct responses in a row and changed to a new puzzle within one minute of work. Figure 2 illustrates the student's behavior on a puzzle involving Basic series.



Note. Student responses are underlined.

Figure 2. Student's behavior on Basic series in first hour.

In all figures, student responses are underlined to distinguish them from computer actions. In Figure 2, line 1 shows the first five terms of the series as they were displayed for the student. Each time the student makes a correct guess, his guess is preceded by "***," as represented in lines 2, 4, and 6 of Figure 2. This new, correct term is then appended to the previous terms. The revised displays of the series after each correct guess are contained in lines 3, 5, and 7. The student chose to change problems after his third correct response, which is represented in line 8 by

"CHANGE." In the series illustrated in Figure 2, the student recognized two separate subseries comprising the main series. He identified and extended them as COUNT and ALPHA series without error. The student exhibited similar behavior whenever he encountered these Basic series, as well as BCOUNT and BALPHA series, beginning at random letters or numbers, throughout his first hour of work. These Basic series were extended in different contexts, including the interspersed terms in Figure 2 and concatenated terms such as "1ØØU, 99T, 98S, 97R, ..."

Basic series were also applied as hypotheses in more complex series. For example, given the display "C, I, W, B, I, X, . . .," the student guessed "A," an incorrect guess. It is inferred that he analyzed the series as containing three subseries: "C, B, . . ."; "I, I, . . ."; and "W, X, ..." The seventh term of the main series is the third term of the first subseries. The guess, "A," then represents a hypothesis that the first subseries is BALPHA beginning at "C" (C, B, A, . . .).

After the first hour on Series, the student continued to recognize and extend the ALPHA, BALPHA, COUNT, and BCOUNT Basic series.

During his first hour on Series, the student recognized the identity relationship between terms and solved REPEAT series. He was successful in solving all REPEAT series he encountered. Those series included series with from 2 to 12 terms in a period. For instance, the first REPEAT series he saw repeated after 12 terms. The student used "NEXT" nine times in a row until two terms appeared which were identical to the first two terms of the series. He then typed enough correct items to solve the series. It is inferred from this behavior that the student recognized the identity relationship and proceeded to extend the series by entering subsequent terms as they appeared earlier in the series. A series containing REPEAT as a subseries was extended and solved without error (9, 1Ø, 9, 11, 9, . . .). The student also solved REPEAT series with six terms in a period (C, I, W, B, I, X, C, I, W, . . .; and I, H, V, H, H, W, I, H, V, . . .).

In his second hour of work, the student extended a series requiring the repetition of a three-element term. Each term was formed by the concatenation of three REPEAT subseries: "I, I, I, . . ."; "85, 85, 85, . . ."; and "O, O, O, . . ."; giving the final appearance, "I85O, I85O, I85O, ..." He made no errors extending both a single-element repetition (16, 16, 16, . . .) and a Concatenate series with a subseries of two repeated symbols (-Q, +R, -S, +T, . . .). An Intersperse series similar to those solved in his first hour of work was extended with no errors (1, X, 2, X, 3, . . .).

The student continued to solve REPEAT series when he encountered them in hours three and four. In addition, he incorrectly hypothesized the existence of REPEAT patterns in seven series. Typical behavior in the student's third hour on Series with a REPEAT pattern is shown in Figure 3.

1. O, B, P, O, B
2. P
3. NEXT
4. O, B, P, O, B, O,
5. B
6. P
7. ••• O
8. O, B, P, O, B, O, O
9. ••• B
10. O, B, P, O, B, O, O, B,
11. ••• P
12. O, B, P, O, B, O, O, B, P,
13. ••• O
14. O, B, P, O, B, O, O, B, P, O,
15. ••• B
16. O, B, P, O, B, O, O, B, P, O, B,
17. ••• O
18. O, B, P, O, B, O, O, B, P, O, B, O,
19. ••• O
20. O, B, P, O, B, O, O, B, P, O, B, O, O,
21. CHANGE

[Puzzle 87, Difficulty 17]

Figure 3. Student's behavior on REPEAT series in third hour.

All student responses are underlined, while computer actions are not. Correct responses are preceded by "***," as described for Figure 2. In addition, Figure 3 contains incorrect guesses. These are the underlined terms in lines 2, 5, and 6. It is inferred from his first guess, "P" (line 2), that the student initially hypothesized a REPEAT series with three terms (O, B, P) in the period. When this hypothesis proved incorrect, he typed "NEXT" for additional information (line 3). Both previous times that "O" appeared in the series, it was followed by "B." The subsequent guess of "B" implies that the student hypothesized another instance of the "O, B" sequence (line 5). It is inferred that the student's next two guesses, "P" and "O" (lines 6 and 7), were hypotheses that the series consisted of the letters "O," "B," and "P" only, in some pattern. It is reasonable to assume that "P" was guessed before "O" because there was no previous instance of two identical letters in a row. When the "O" proved correct, the student hypothesized that the series was a REPEAT series with six terms (O, B, P, O, B, O) in the period and correctly extended the series by seven additional terms (lines 8 through 20). This protocol provides evidence of the student's ability to recognize the identity relationship and to hypothesize various REPEAT patterns in a series.

Concatenate Series

Concatenate series are those in which terms from two or more subseries are written next to each other to form a single term of the main series. When the subseries contain different symbol sets, usually numbers in one and letters in the other, the contrast between subseries is high and the Concatenate is easier to solve. When the contrast is low or the subseries are more complex than Basic series, Concatenate series are harder to solve.

Throughout his Series history, the student made no errors extending Concatenate series composed of the contrasting Basic subseries: COUNT

or BCOUNT with ALPHA or BALPHA; and REPEAT of "-1, +" with ALPHA or COUNT. Series solved without error included "1A, 2B, 3C, . . ."; 100U, 99T, 98S, . . ."; and "-1, +2, -3, ..." In this third hour, the student encountered two Concatenate series that presented some difficulty for him. The first consisted of two REPEAT subseries with different length periods: REPEAT of "R, S, T" and REPEAT of "3, 4, 5, 6, 7." His solution of this series appears in Figure 4. From the student's repeated guess, "T8" in lines 2 and 3, he clearly recognized the repeated "R, S, T" subseries and hypothesized the second subseries to be a COUNT series. He often guessed the same term twice whan a hypothesis was strongly implied.

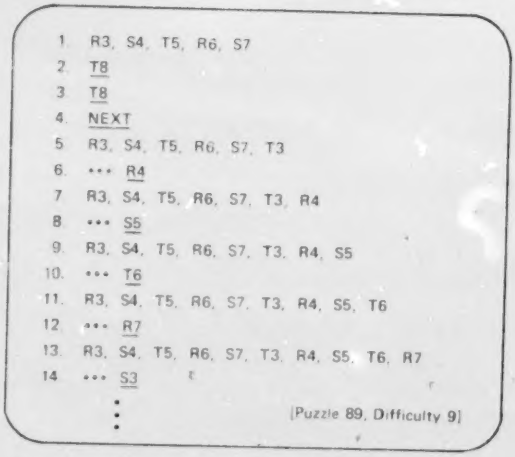


Figure 4. Student's behavior on Concatenate series in third hour.

The student asked for the next term (line 4), which was "T3." This revealed the student's hypothesis about the first subseries to be correct and the second subseries to be a REPEAT pattern as well. The student proceeded to extend the series successfully, entering the correct term, "S3,"

at line 14 when the second subseries started a new period. He continued entering correct terms through a third start of that pattern before typing "CHANGE."

A Concatenate series that the student did not solve contained a Skip ALPHA subseries (every third letter) and a BALPHA subseries (WJ, XI, CH, FG, . . .). The student's guesses always contained a correct extension of the second subseries, but he could not extend the first.

Skip Series

Skip series exhibit wide variety because the pattern by which skipping occurs within one subseries is controlled by a second numeric subseries of arbitrary complexity. The first subseries can be either alphabetic or numeric; the magnitude of numbers involved can vary widely; the direction of skipping can be either forward or backward; the number of terms in a period can be one, two, three, or even higher; and the amount to skip can change significantly. Each of these factors contributes to the difficulty of the solution. The student's work will be traced across time in several categories suggested by the distinctions drawn here.

In alphabetic Skip series, the student made no errors when every other letter was skipped, as in "S, U, W, Y, A, ..." He had more difficulty with skipping two letters. He first encountered a series of this type as a subseries of a Concatenate series. His guesses indicated that his hypothesis was a skip of only one letter, and he could not solve it at that point. Two puzzles later (see Figure 5), the student did solve this series when he encountered it out of the Concatenate context. His first guess, "A" in line 2, implies a hypothesis that the series skipped only one letter, but he then extended the series successfully. The next time he saw this type of series, the student extended it without error.

1. M, P, S, V, Y,
2. A
3. ••• B
4. M, P, S, V, Y, B,
5. ••• E
6. M, P, S, V, Y, B, E,
7. ••• H
8. M, P, S, V, Y, B, E, H,
9. ••• K
10. M, P, S, V, Y, B, E, H, K,
11. CHANGE

(Puzzle 32, Difficulty 23)

Figure 5. Student's behavior on alphabetic Skip series in third hour.

During his first hour of work, the student made no errors solving numeric Skip series that increased or decreased by a constant amount. These included series starting at from 1 to 94, with differences between terms from 3 to 11Ø, e.g., "3, 6, 9, 12, . . .," and "64, 131, 198, 265, ..." One puzzle solved without error was based on backward counting: "94, 9Ø, 86, 82, ..." A series in which each term was doubled to form the next was extended as well (1, 2, 4, 8, 16, . . .).

In his second hour of work, the student solved a BCOUNT Skip series with a difference of 54 between terms without error: "4989, 4935, 4881, 4827, ..." He also solved a series beginning at 484 with a difference of 1Ø between terms. However, he made arithmetic errors on some series he previously solved without error, series which started at 2 and 3, increasing by 8 and 6, respectively. He also made arithmetic errors on a series beginning at 5955 and decreasing by 75.

Figure 6 shows his behavior on another series which was also at a difficulty level he previously solved. Instead of consistently adding the

correct difference of 8 between terms, the student added 5, 7, 17, and 18 in lines 4, 8, 11, and 12 (line 5 contains the student's typographical error).

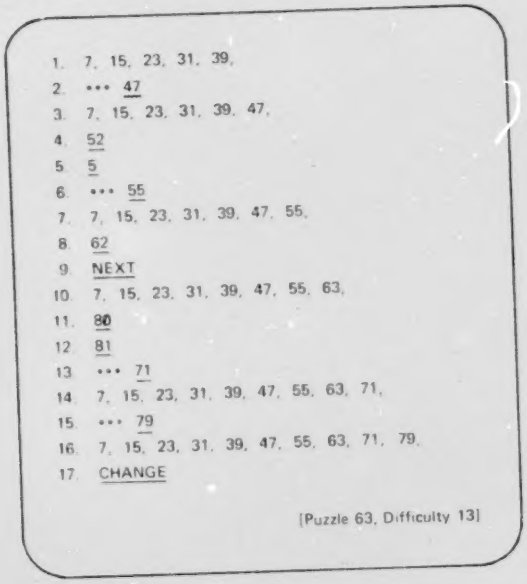


Figure 6. Student's behavior on numeric Skip series with constant increment in second hour.

In his third hour, the student continued to extend without error Skip series with constant differences beginning at numbers 1 to 1Ø with differences of magnitude 1 to 1Ø. He also made no errors on Skip series of BCOUNT starting at 226 and 884, with differences 8 and 14, respectively.

During his final hour on Series, the student made no errors solving series beginning at 6 and increasing by 6 and beginning at 5Ø and increasing by 1ØØ. He did make some arithmetic errors on, but solved, a series starting at 72 with a difference of 16 between terms.

A Skip series which the student did not solve is shown in Figure 7. The student never added the correct difference of 8 to a previous term; instead, his guesses were increments of 18, 10, and 2 (lines 2, 3, and 6).

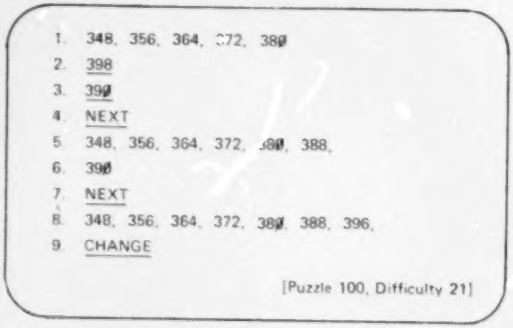


Figure 7. Student's behavior on numeric Skip series with constant increment in fourth hour.

Numeric Skip series with period two skip by a repeated pattern of two numbers, such as "2, 3, 2, 3, 2, 3, . . ." or "3, -1, 3, -1, 3, ..." The sizes of the initial number of the two repeating numbers determine the difficulty of the series. In his first hour of work, the student made no errors extending numeric Skip series of period two that started at numbers less than 10 and increased or decreased by differences of 5 or less (both differences of the same sign), such as "6, 7, 9, 10, 12, ..." All other such series throughout his history were also solved without error.

In his first hour, the student made some errors on, but solved, all numeric Skip series of period two which started at numbers up to 100 and generated terms with any combination of positive and negative differences of 7 or less. Typical behavior on such series appears in Figure 8. This series begins at 100 and decreases by alternating values of 1 and 2. The

student entered two correct terms (lines 2 and 4), decreasing the previous term by 1 and 2, but then decreased by 2 again out of the proper sequence. Evidence such as this implies the student's understanding of the conceptual principle of Skip series with period two, but a tendency to lose track of his position within the period. During the rest of his Series history, the student solved most such series without error.

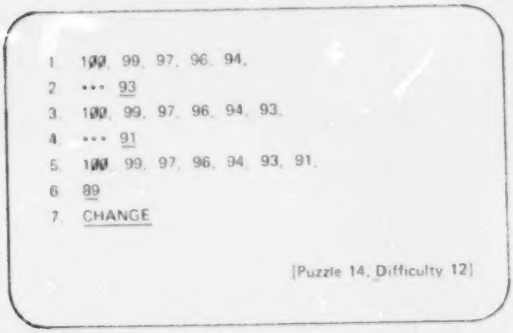


Figure 8. Student's behavior on numeric Skip series with period two in first hour.

The student did have difficulty with one series in the fourth hour, as illustrated in Figure 9. Although he had solved puzzles similar to this without error, he had difficulty keeping track of the period here. This series decreased by alternating values of 1 and 3. The student erred by using the wrong difference in lines 2 and 8 and by using the wrong sign of difference in line 3, but he then entered five correct responses in a row, demonstrating a correct alternation of difference values.

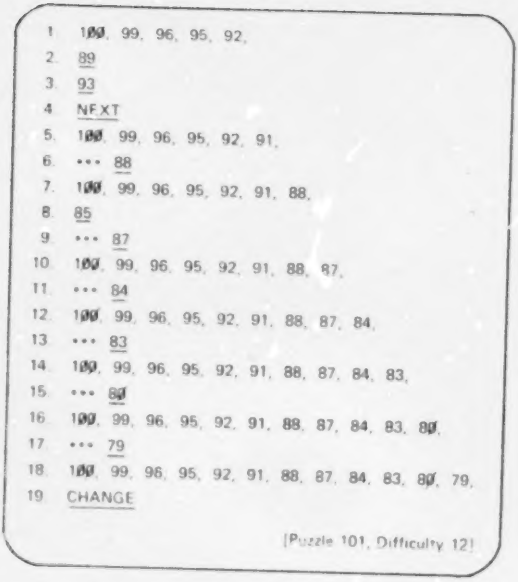


Figure 9. Student's behavior on numeric Skip series with period two in the fourth hour.

In his second and third hours, the student encountered, but did not solve, numeric Skip series of period two that started with numbers of magnitude up to 1ØØ. For example, one series started at 6 and alternately increased by values of 18 and 39. He used the difference value 18 to enter one correct term, but tried the same value again for the next term. In his fourth hour, he did solve without error a series beginning at 473 with differences between terms of +6 and -7. That series, "473, 479, 472, 478, 471, 477, . . . ," may have been solved as an Intersperse of two BCOUNT subseries.

Numeric Skip series of period two with multiplicative differences between terms were difficult for the student. He solved one such series

without error in his first hour of work (1, 2, 5, 10, 13, . . .) in which terms were generated by alternately multiplying by 2 and adding 3. Although seven were attempted, no other series with multiplicative differences were solved in the remainder of the student's Series work. The student typically saw differences as additive instead of multiplicative in these series, as illustrated in Figure 10.

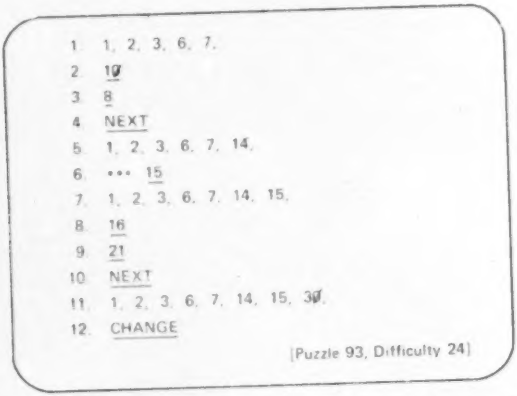


Figure 10. Student's behavior on numeric Skip series with period two requiring multiplication in third hour.

In the series in Figure 10, terms are generated by alternately multiplying by 2 and adding 1. The resulting differences between the five terms originally displayed for the student (line 1) are all either 1 or 3. The student's first two guesses, "10" and "8" (lines 2 and 3), were increments from the last term, "7," of 3 and 1, respectively. Since these guesses were incorrect, the student asked for additional information. He guessed "15" (line 6), an increment of 1 from the new term, which was correct. His next guess was also an increment of 1 (line 8), but a multiplication operation was required, and his guess was incorrect. The subsequent guess of "21" (line 9) was most likely an increment of 7 from 14, indicating

his perception of the "7, 14" sequence as an addition of 7. With the additional term provided by his request for "NEXT," he still did not perceive the multiplicative relationships between 3 and 6, 7 and 14, and 15 and 30, and chose to "CHANGE."

Numeric Skip series with period three use three different increment values and are more complex than those with period two. In his first hour, the student encountered the only period three Skip series he ever solved. The series started at 9 and increased by three positive differences, 1, 2, and 3. He solved this series without error (9, 10, 12, 15, 16, 18, 21, . . .). This repeating pattern of differences (1, 2, 3, 1, 2, 3, 1, . . .) is among the simplest possible. An earlier puzzle of differences was "1, -2, 3, 1, -2, 3, 1, ..." The student made errors of sign and magnitude and entered only one correct guess before changing to a new series.

In his second hour of work, the student partially solved a series with terms and differences of greater magnitude than those in his first hour. This series is shown in Figure 11. This series has differences between terms of +14, -5, and +21. The student's guess of "62" (line 4) was an

1. 12, 26, 21, 42, 56,
2. NEXT
3. 12, 26, 21, 42, 56, 51,
4. 62
5. ••• 72
6. 12, 26, 21, 42, 56, 51, 72,
7. ••• 86
8. 12, 26, 21, 42, 56, 51, 72, 86,
9. CHANGE

[Puzzle 36, Difficulty 33]

Figure 11. Student's behavior on numeric Skip series with period three in second hour.

increment of 11 from the previous term. This was most likely a guess based on an arithmetic error, for the student then added the correct increment of 21. He again incremented correctly by 14 before typing "CHANGE." The student did not enter enough terms for complete judgment of his understanding of this series. He could not solve the only other Skip series of period three he encountered, which started at 494 and had differences of +15, -5, and -8 between terms.

Intersperse Series

Intersperse series are formed by taking individual terms of the main series alternately from a set of two or more subseries. The difficulty of an Intersperse series is controlled by the contrast between the subseries and the difficulty of the component subseries themselves. Throughout his four hours, the student made only one incorrect guess while solving Intersperse series of period two, with one subseries alphabetic and the other numeric. A typical example is "X, 6, Y, 7, Z, ..." The subseries used included REPEAT, COUNT, and ALPHA. The student made no errors extending a series with two numeric subseries, one REPEAT and one COUNT (9, 10, 9, 11, 9, 12, . . .).

A series with three interspersed alphabetic subseries was encountered by the student. His work on this series appears in Figure 12. The series in Figure 12 consists of three subseries: ALPHA beginning at "E," ALPHA beginning at "V," and REPEAT of the letter "M." It may be inferred from the student's guess "N" (line 2) that he recognized the presence of three subseries and hypothesized that if the first two were ALPHA, the third was as well. When this proved incorrect, he asked for more information. He then extended the first two subseries correctly (lines 5 and 7) and typed "CHANGE." It is likely that the student understood the entire series, but the protocol does not contain an extension of the third subseries.

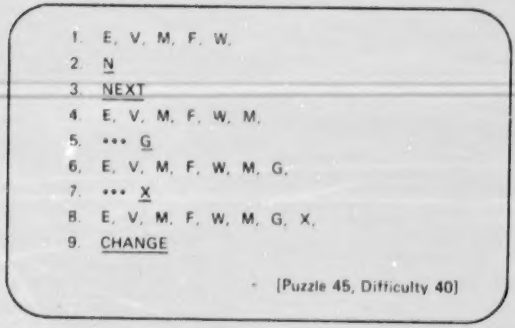


Figure 12. Student's behavior on alphabetic Intersperse series with period three in second hour.

Intersperse series with numeric Skip subseries presented greater difficulty for the student; he did not solve any such series he encountered. Typical student behavior on an Intersperse of two numeric Skip series appears in Figure 13. The first subseries of the Intersperse series is a Skip series beginning at 68 and increasing constantly by 4; the second begins at 48 and increases constantly by 3. The student's first guess, "81" (line 2), is an increment of 5 from the last term. This difference may have been extracted from the difference between the last digits of the previous two terms, 51 and 76. His next guess, "56" (line 3), was a decrement of 2Ø, the first difference in the main series (between 68 and 48); his final guess, "1ØØ" (line 4), was an increment of 24, the second difference in the main series (48 to 72). None of the student's guesses imply a recognition of an Intersperse pattern. However, when two such numeric Skip series were interspersed with an alphabetic subseries, increasing the contrast among subseries, the student did successfully extend the alphabetic series.

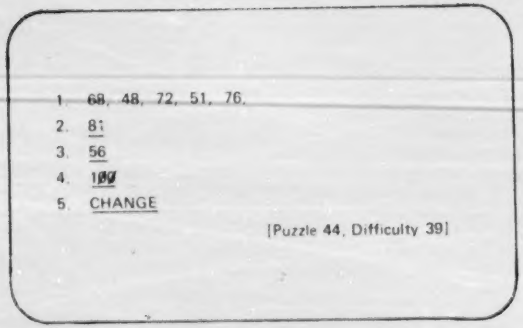


Figure 13. Student's behavior on numeric Intersperse series with Skip subseries in second hour.

Typical student behavior on an intersperse of two alphabetic Skip subseries appears in Figure 14. The two alphabetic subseries are: "T, Y, D, . . ." and "V, Z, D, . . ."; they skip by five and four letters, respectively. The student never saw alphabetic Skip series with such large increments as a main series, so the series in Figure 14 proved difficult.

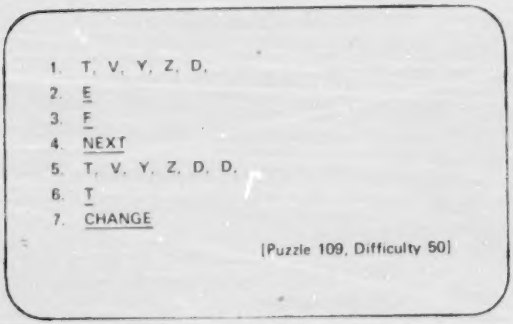


Figure 14. Student's behavior on alphabetic Intersperse series with Skip subseries in fourth hour.

His first two guesses, "E" and "F," are local increases from the fifth term "D." The use of "NEXT" is excellent. The student's next guess, "T," stems from a hypothesis that the series repeats from its first term. Throughout his work on the series, the student shows no awareness of the Intersperse pattern in the series. Two other Intersperse series presented similar difficulty for the student.

The student made some progress toward solving a period four series containing REPEAT and Skip subseries, as illustrated in Figure 15. The series has many confusing terms, but can be decomposed into four simple subseries that alternate. The first repeats 6; the second repeats 7; the third skips counting by a constant 7 (7, 14, 21, 28, . . .); and the fourth repeats 7. Eight of the twelve guesses before line 24 in Figure 15 were "6" and "7," the numbers that are repeated in three of the subseries and are correct for nine of the first ten terms. The remaining four guesses are governed by more local patterns. The guess of "8" in line 6 is generated by a counting hypothesis beginning with the subseries "6, 7, ..." A similar counting appears with "15" in line 10. Here the hypothesis is more complex, involving the fact that counting occurs elsewhere. Some awareness of the periodicity may therefore be in evidence as early as line 10. The guess of "14" in line 11 recognizes that terms sometimes are repeated after themselves. Finally, the guess of "14" in line 20 recognizes the local subseries "6, 7, 6, 14, 6, 7, 6, . . ." as a possible repeating pattern with period four. After line 22, four of the next five guesses are correct. The error comes from anticipating "28" too early, losing track of which subseries is in control. The student stopped working at this point when his understanding is still in doubt, but he has made excellent progress on the series.

1. 6, 7, 7, 7, 6,
2. 6
3. ••• 7
4. 8, 7, 7, 7, 6, 7,
5. 7
6. 8
7. 6
8. NEXT
9. 6, 7, 7, 7, 6, 7, 14,
10. 15
11. 14
12. NEXT
13. 6, 7, 7, 7, 6, 7, 14, 7,
14. 7
15. NEXT
16. 6, 7, 7, 7, 6, 7, 14, 7, 6,
17. ••• 7
18. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7,
19. 7
20. 14
21. 6
22. NEXT
23. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7, 21,
24. ••• 7
25. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7, 21, 7,
26. ••• 6
27. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7, 21, 7, 6,
28. 28
29. ••• 7
30. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7, 21, 7, 6, 7,
31. ••• 28
32. 6, 7, 7, 7, 6, 7, 14, 7, 6, 7, 21, 7, 6, 7, 28,
33. CHANGE

[Puzzle 84, Difficulty 21]

Figure 15. Student's behavior on numeric Intersperse series with period four in third hour.

Combined Types

The increasing increment series are related to Skip series. Instead of increasing by a constant amount or by a repeating pattern of differences, the amount of increase is itself a Skip series. For example, in the increasing increment series "1, 6, 13, 22, 33, 46, 61, 78, . . .," the subseries by which terms increase is "5, 7, 9, 11, 13, 15, 17, ..."

The student encountered, but could not solve, three increasing increment series in his first two hours on Series. Of these, the most interesting is illustrated in Figure 16. The increments in the puzzle increase by doubling, "5, 10, 20, 40, 80, 160, 320, ..." The student's guesses show little awareness of the pattern. Some information has been assimilated, though, for most of the student's guesses end in the digit "6," as do all terms.

1. 1, 6, 16, 36, 76,
2. 126
3. 1
4. NEXT
5. 1, 6, 16, 36, 76, 156,
6. 205
7. NEXT
8. 1, 6, 16, 36, 76, 156, 316,
9. 436
10. 336
11. NEXT
12. 1, 6, 16, 36, 76, 156, 316, 636,
13. 976
14. 876
15. CHANGE

[Puzzle 46, Difficulty 41]

Figure 16. Student's behavior on increasing increment series with multiplicative Skip subseries in second hour.

The first guess, "126" (line 2), is an increase of 5Ø from the previous term where the increment was 4Ø. Only at line 9, when he guesses "436," 34Ø above the previous term where the previous increment was 32Ø, does the student show similar awareness of the increasing increment.

In his fourth hour, the student again encountered an increasing increment series. The series started at Ø and had the difference series "5, 1Ø, 15, 2Ø, 25, . . ." (see Figure 17). It is clear from the student's first three responses (lines 2, 4, and 6) that he analyzed the progression of differences correctly and incremented by 25, 3Ø, and 35, as required. He lost track of the differences and incremented by 35 to guess "175" in line 8, but he immediately corrected himself. There is no obvious explanation for the error in line 11, but the student entered the next term correctly and did solve this increasing increment series. Learning occurred in this class of series.

1. Ø, 5, 15, 3Ø, 5Ø,
2. ••• 75
3. Ø, 5, 15, 3Ø, 5Ø, 75,
4. ••• 1Ø5
5. Ø, 5, 15, 3Ø, 5Ø, 75, 1Ø5,
6. ••• 14Ø
7. Ø, 5, 15, 3Ø, 5Ø, 75, 1Ø5, 14Ø
8. 175
9. ••• 18Ø
10. Ø, 5, 15, 3Ø, 5Ø, 75, 1Ø5, 14Ø, 18Ø,
11. 126
12. ••• 225
13. Ø, 5, 15, 3Ø, 5Ø, 75, 1Ø5, 14Ø, 18Ø, 225,
14. CHANGE

[Puzzle 106, Difficulty 24]

Figure 17. Student's behavior on increasing increment series with additive subseries in fourth hour.

Series with increasing period, a second combined type, are those in which the period gets longer as the series progresses, for example, "1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ..." The student encountered one such series in his first hour of work (6, 7, 7, 8, 8, 8, 9, . . .). Given the first five terms of this series, the student guessed "9" twice and changed puzzles.

In his fourth hour, the student again encountered an increasing period series. His solution appears in Figure 18. The student's first guess, "F" (line 2), was a hypothesis that each letter repeated twice, disregarding the first, single, letter. Notice the similarity to his guess of "9" on the puzzle just described. His second guess, "C" (line 3), is generated by a hypothesis of a REPEAT of all five terms. He then guessed "E" (line 4) and "F" (line 6) and extended the series successfully for eight additional terms beyond those shown in Figure 18. Learning has occurred on increasing period type series.

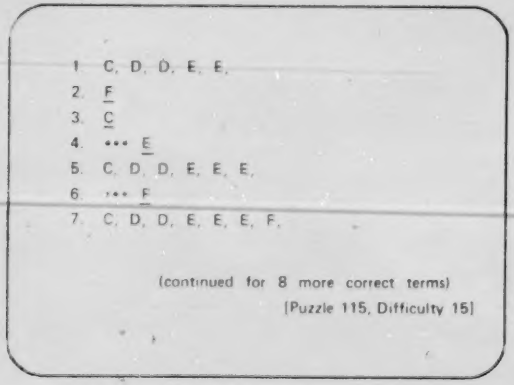


Figure 18. Student's behavior on alphabetic increasing period series in fourth hour.

Summary of Series-Solving Skill Development

Protocol analysis provides evidence of the student's entering knowledge of the Basic series COUNT, BCOUNT, ALPHA, BALPHA, and REPEAT. These Basic series were recognized and extended correctly when encountered alone or as subseries of Intersperse and Concatenate series throughout the student's history.

The student entered with the ability to concatenate two subseries to extend main series; he successfully extended all Concatenate series encountered which were composed of contrasting Basic series (letters with numbers, or numbers with the signs "+" or "-"). Evidence of learning appears in one of these series with which the student had some difficulty (see Figure 4). It is clear from this protocol that the student learned to keep track of two REPEAT subseries of different lengths and to concatenate them according to his current position in each repeating cycle. This learning represents both a specific series-solving skill and a more general problem-solving skill; the student has decomposed the problem, solved each subpart, and recombined the separate solutions to produce a solution of the full problem.

Only one Concatenate series remained ultimately unsolved in the student's history. Since one subseries was beyond his series-solving repertoire at that time, this puzzle was too difficult for the student.

The majority of series the student saw were of the Skip type. These varied widely in difficulty depending on factors discussed in the section on Skip series, and were either numeric or alphabetic. Alphabetic series that skipped every other letter provided no difficulty, but the student struggled to his eventual solution of series that skipped two letters between terms (see Figure 5).

Numeric Skip series with a period of one were solved throughout the student's Series experience. Arithmetic errors were made in adding or subtracting the constant differences between terms in these series, but the student understood the concept involved. Series solved started with numbers between 1 and 5955 and increased or decreased by values from 3 to 100.

Numeric Skip series with a period of two skip by a repeated pattern of two numbers that are either both positive, both negative, or of mixed signs. The student solved almost every series of this type that he encountered. Evidence implies that the student understood the conceptual basis of the Skip series with period two, although he often made arithmetic errors and occasionally lost track of his position within the period of some series. The student's main difficulty with these series arose when difference values exceeded 20.

Multiplicative Skip series, in which the terms are alternately multiplied by a number and either increased or decreased by another number, proved difficult for the student. Although he solved one such series in his first hour of work, the student typically saw differences as additive instead of multiplicative.

One numeric Skip series with a period of three was solved by the student. This series started at 9 and increased by the repeating pattern of "1, 2, 3, 1, 2, 3, 1, ..." Progress was made toward solution of a series with terms and differences of greater magnitude and mixed signs, but the student stopped too soon for complete judgment of his understanding. Two period three Skip series were not solved. Evidence for the student's understanding is inconclusive.

Intersperse series of period two with contrasting Basic series (one alphabetic and one numeric subseries) were solved consistently by the

student throughout his Series experience. He was at least partially successful on all series with two numeric subseries or three alphabetic subseries but had difficulty with interspersed Skip subseries of low contrast. It appeared that when the interspersion was easily perceived, the student solved each subseries and combined them to solve the main series. However, when the interspersion was not suggested by the immediate appearance of the series, the student did not hypothesize the existence of interspersed subseries.

During his Series experience, the student learned how to solve series with increasing increments. In the three series of this type seen in the student's second hour of work, only minor evidence of awareness of the increasing increment was found. A fourth series, seen in the student's fourth hour of work, was solved using the correct progression of differences.

A second combined type, series with increasing period, was not solved by the student when first encountered. In his second exposure to this type of series, the student worked out the correct solution. Learning occurred on increasing period series.

Throughout the discussion of series-solving skills, we have interpreted the data as conservatively as possible. Thus, if no errors were made on a series, we inferred that the student knew how to do that series when he began his work on the program. It seems likely that a less conservative interpretation would be justified; some learning that occurred during work on the preceding series helped in solving the new one. There is no way to determine the facts from the data in hand, and we choose to present the data cautiously.

Development of Problem-Solving Skills

This section describes the student's development of problem-solving skills, including: (a) testing of hypotheses, (b) use of feedback from guesses, (c) gathering of additional data when needed, (d) consideration of all of the data available, (e) choice of appropriate time to stop work on a series, (f) actions after groups of correct guesses, (g) actions after incorrect guesses, and (h) use of overall results on series for subsequent choice of new difficulty level.

First Hour

Substantial changes in the student's problem-solving behavior occurred within the first hour of work. Overall he chose difficulties increasing steadily from 1 to 31 with one exploration to difficulty 50. This information is summarized in Table 2 along with similar data for the other three hours.

Table 2

Percentage of Puzzles Selected in Each Difficulty Range for Each Hour

	Hour			
Difficulty Range	1[a]	2[b]	3[c]	4[d]
1 - 10	32	30	39	33
11 - 20	29	13	26	21
21 - 30	32	7	19	29
31 - 40	3	33	3	4
41 - 50	3	17	13	12
	$^{a}\underline{n} = 34$	$^{b}\underline{n} = 30$	$^{c}\underline{n} = 31$	$^{d}\underline{n} = 24$

In his first hour, the student passed 22 of the 34 series he saw with no errors at all, and passed 6 of the remaining 12 after minor errors. Therefore, his gradual increase of difficulty may represent a too conservative use of the feedback on his overall results. On the 22 series solved without error, he chose appropriate times to change problems after he knew the solution. The 12 series with which he had difficulty provide insight into his other problem-solving skills.

On a series which presented difficulty in the early part of the hour's work, the student typically behaved as illustrated in Figure 19. After making two incorrect guesses (lines 2 and 3), the student changed to a new series. The student was testing a reasonable hypothesis but did not use the feedback that his first guess was incorrect; he made the same guess twice. He did not attempt to gather additional data which may have led to a solution and did not consider all of the information available, that is, he disregarded the fact that the first term appeared only once. His problem-solving repertoire seems limited.

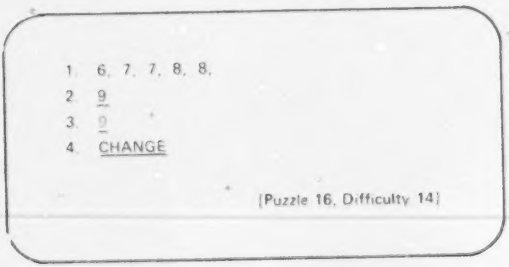


Figure 19. Poor use of feedback in first hour.

Another example of the student's early repertoire is illustrated in Figure 20. The student entered one correct response (line 2), made three

plausible, but incorrect, guesses, and changed. He did not request additional data. The student changed without asking for the additional data that might have been helpful in two other series during the first hour. We infer that the student did not recognize the utility of typing "NEXT" to get help until after Puzzle 24, although he had used it in Puzzle 2 where he typed "NEXT" nine times in a row until he noticed that the last two terms were repetitions of the first two. He then proceeded to extend the REPEAT pattern. This was the only use of "NEXT" until 52 minutes into his first hour.

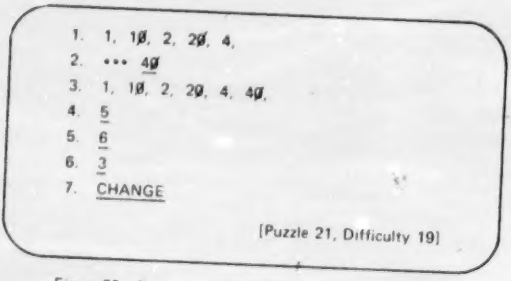


Figure 20. Failure to seek additional data in first hour.

Figure 21 illustrates a new step in the development of the student's skills. The student was faced with three interspersed subseries, one alphabetic and two numeric series. He tested two hypotheses and then typed "NEXT" (line 4). This was an appropriate time to ask for new data since feedback indicated his two hypotheses were incorrect. He correctly extended the alphabetic subseries (line 6) but then typed "CHANGE." He did not do any more work toward extension of the two numeric subseries, once again stopping before trying every possible strategy toward solution.

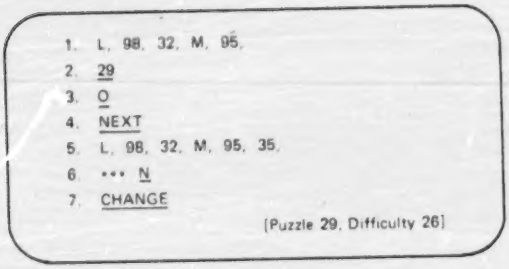


Figure 21. Seeking additional data in first hour.

A more effective strategy is seen in Figure 22 where the student asked for an additional term (line 2) as soon as he saw the original display in line 1. We infer from his guess of "A" (line 4) that he hypothesized the series was comprised of three subseries, the first of which (terms 1, 4, and 7) was BALPHA beginning at "C." He sought more information after receiving feedback indicating that his hypothesis was incorrect. This new term, "C" (line 6), revealed the possibility of a REPEAT pattern, and the student hypothesized the next term to be "I" (line 7). This was correct and the student proceeded to use all of the data available to him to extend the series for five correct terms in a row. He chose to stop when he was convinced that he thoroughly understood the series. His next puzzle was also a REPEAT series with six terms in a period. He used "NEXT" twice to gather more information and extended the series successfully for seven correct terms in a row.

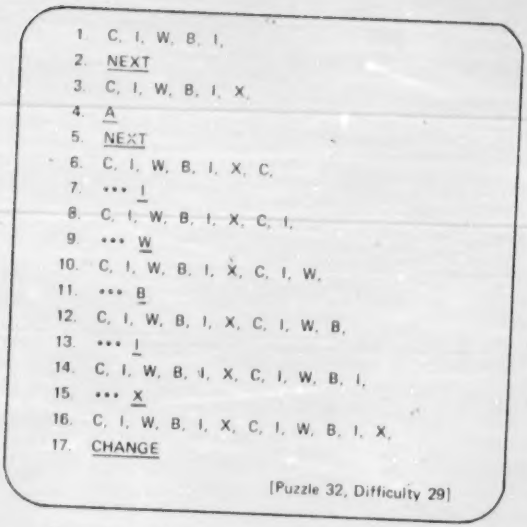

1. C, I, W, B, I,
2. NEXT
3. C, I, W, B, I, X,
4. A
5. NEXT
6. C, I, W, B, I, X, C,
7. ••• I
8. C, I, W, B, I, X, C, I,
9. ••• W
10. C, I, W, B, I, X, C, I, W,
11. ••• B
12. C, I, W, B, I, X, C, I, W, B,
13. ••• I
14. C, I, W, B, I, X, C, I, W, B, I,
15. ••• X
16. C, I, W, B, I, X, C, I, W, B, I, X,
17. CHANGE
[Puzzle 32, Difficulty 29]

Figure 22. Seeking enough additional data in first hour.

Summarizing the first hour's work, the student was seen to create and test reasonable hypotheses. However, in those cases when his hypothesis proved false, he was unable to generate an alternate hypothesis. He used feedback from guesses to decide when to change puzzles. For most of the hour, three or four correct guesses or two incorrect guesses led to a change. During the last few minutes of the hour, wrong guesses led to gathering new data, a much more productive strategy. The choice of new difficulty levels followed a simple rule: Increase difficulty by one each time. This rule led to too slow an increase most of the time.

Second Hour

In his second hour of work, the student chose series of greater difficulty than in his first hour. The distribution of difficulty in the second hour is given in Table 2.

The student's behavior on those series which he failed to solve falls into three categories: (a) testing hypotheses based on given data only; (b) accumulating additional data and testing hypotheses but failing to enter any correct responses, and (c) accumulating additional data and partially solving the series (some correct guesses).

There were just two series in this hour's work on which the student had difficulty and did not use "NEXT." On one of these, the student tested only incorrect hypotheses before changing puzzles, as illustrated in Figure 13. The puzzle in Figure 13 is an Intersperse series comprised of two Skip subseries. This type of series was beyond the student's level of competence and was never solved during his Series experience. Those hypotheses tested in lines 2, 3, and 4 were attempts to extend the series by using differences between terms of the main series. No awareness of the interspersed series is implied, and it is likely that the student used the only series-solving strategy in his current repertoire to make those guesses. Additional data might therefore have been of no assistance in this particular puzzle. Conversely, had the student persisted by asking for new terms and testing hypotheses, it is possible that he may have eventually recognized the intersperse relationship.

Another series in which additional information would more likely have led to a solution is illustrated in Figure 23. The puzzle is an Intersperse series. The student's correct extension of the second subseries in line 2 implies an awareness of the Intersperse pattern. His guess "O" (line 4) reinforces the inference that he recognized two separate series and had only to deduce the precise Skip value in the first subseries. A request for more terms would have been a productive strategy here; the student changed too soon.

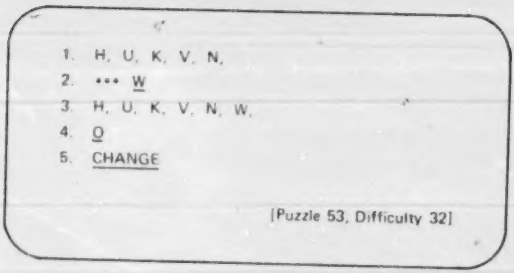


Figure 23. Changing puzzles too soon in second hour.

Additional data was requested by the student in all other series with which he had difficulty during the second hour of work. On several of these series, he did succeed in making some correct guesses. One such series is shown in Figure 11. The puzzle in Figure 11 is a Skip series of period three with differences between terms of +14, -5, and +21. The student asked for another term immediately (line 2), and made one incorrect guess before proceeding to extend the series with two correct terms. He considered all available data in order to enter these terms with the increments of 21 and 14 in proper sequence. He stopped working before entering a third term which would have confirmed a total solution of the series, but he may have considered the series solved.

Another appropriate use of "NEXT" is shown in Figure 12. The student guessed "N" (line 2) hypothesizing the third subseries to be ALPHA beginning at "M." Feedback indicated this hypothesis was incorrect, and the student requested another term. He then extended the other two subseries correctly. Once again, he stopped working without entering a correct third term, but he was probably convinced of his own understanding of the series when he stopped. Four other series included requests for additional data when needed and demonstrated the student making some progress toward solution.

The student exercised problem-solving skills on several series during his second hour on which he had no success. One such series appears in Figure 24. This series is one with an increasing increment, a type with which the student had much difficulty. In this instance, the student tested three hypotheses (lines 2, 3, and 4) and asked for more data when feedback indicated they were incorrect (line 5). He made two more incorrect guesses (lines 7 and 8) and used "NEXT" twice to gather more terms for consideration (lines 9 and 11). This new information was apparently of no help. The student used whatever series-solving and problem-solving strategies were available to him, and when he could still make no progress, he appropriately chose to go to a new puzzle. In four other series, the student made no progress toward solution but chose to stop working at appropriate times.

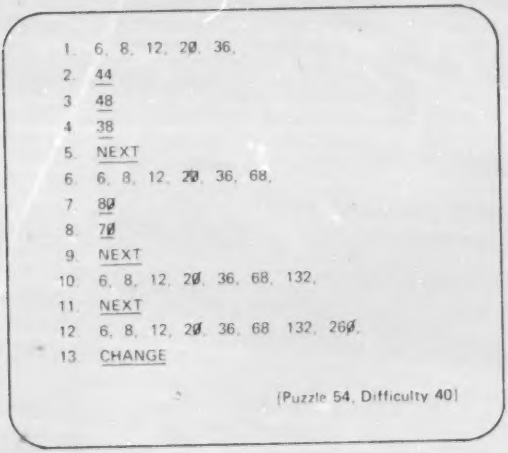


Figure 24. Good problem solving in second hour.

During this second hour, the student solved many series without errors or with minor arithmetic errors. His behavior changed considerably from such series in his first hour. Instead of entering his usual 3 to 5 correct terms, as in the first hour, the student entered from 1 to 28 correct responses in a row. He typed 6 or more right in a row on more than half of these puzzles. These series were trivial for the student, e.g., ALPHA beginning at A: 28 right in a row; COUNT beginning at 38: 13 right in a row. Such large numbers of correct answers in a row are excessive, and this behavior cannot be considered productive problem solving.

Third Hour

During his third hour on Series, only ten puzzles proved difficult for the student. He tested hypotheses and requested additional data on all ten of these series and succeeded in solving two of them. Of those he could not solve, he did succeed partially on three. Typical third hour behavior on one of these appears in Figure 10. The student tested two hypotheses which proved incorrect (lines 2 and 3) and requested another term (line 4). His next guess (line 6) correctly incremented the new term by 1, but his next two hypotheses were not correct (lines 8 and 9). He again asked for more information but still could not discern a solution and appropriately chose to stop. This pattern of testing hypotheses and asking for "NEXT" is an effective problem-solving tactic which the student used frequently during the third hour. See also his behavior when working on the series in Figure 18.

Figure 25 illustrates some typical third hour behavior. When the student's hypothesis in line 5 proved incorrect, he asked for another term. This was an appropriate time for the request, but the student then stopped working on the series. Here the student changed too soon. He did not persevere on the task long enough to determine how much progress he

might have made toward a solution. The student stopped too soon on four other series during this hour. In contrast, the student stopped at an appropriate time on Puzzle 90; he tested four hypotheses, asking for more data after the first and third, and stopped work only when every series-solving strategy in his repertoire at that time was used.

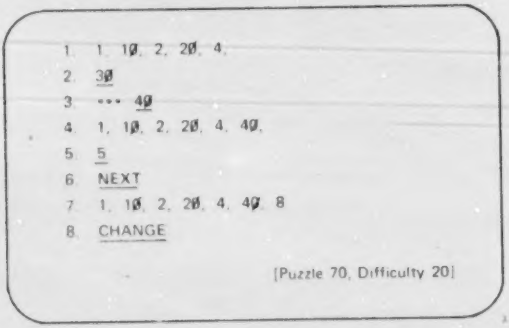


Figure 25. Changing puzzles too soon in third hour.

The two series the student solved with effective problem-solving strategies during his third hour appear in Figure 3 and Figure 4. These both illustrate student testing of hypotheses, use of feedback from incorrect guesses, requests for more information when needed, consideration of all of the available data, and appropriate choice of stopping time when solution was confirmed.

On those puzzles which the student solved without algorithmic difficulty, or with no errors, behavior changed during the third hour's work. Excessive correct terms in a row were entered in the early part of the hour (between 7 and 25 right in a row), but no more than 5 correct responses in a row were entered in the remainder of the hour on this type of puzzle.

Fourth Hour

In his fourth hour on Series, the student worked on only six puzzles which required application of problem-solving skills. One of these was solved and extended successfully with use of feedback from two incorrect hypotheses, as is shown in Figure 18. Figure 7 and Figure 14 illustrate two more examples of student testing of hypotheses and use of "NEXT" but without progress toward solution. Figure 9 contains the last example of student problem solving leading to successful extension of a series. The student tested hypotheses, chose to gather information after receiving feedback regarding these hypotheses, used all available data in his eventual solution, and stopped working when a solution was assured by five right terms in a row.

On the rest of the series of this hour's work, the student made either arithmetic, typographical, or no errors. On three such series, he entered excessive correct terms in a row (53, 21, and 32 terms). On the remaining 13 puzzles, the student stopped working after solution was assured by no more than 7 correct answers in a row.

A final, more global, perspective for viewing the student's problem-solving behavior is to consider his plan for attacking the entire set of available series. The student's use of his overall result on a puzzle for choosing the subsequent difficulty level is one aspect of this behavior.

Series are available at difficulty levels from 1 to 50. During his first hour of work, the student approached the series methodically, requesting difficulty levels in increments of one from his last request. In this way, he requested series from difficulty 1 to 31 in his first hour. He did not solve six puzzles during this time but chose to proceed one difficulty level at a time regardless of the result on any puzzle. Although he was primarily successful with the puzzles seen, the lack of responsiveness to feedback regarding result cannot be considered a constructive problem-solving strategy.

At the beginning of his second hour, the student continued his climb from 32 to 35 but encountered difficulty with each puzzle. As a result, he abandoned his technique of steady incrementation at this point. He began choosing difficulty levels with no clear pattern, still without regard for results on previous puzzles. (The one exception occurred at the end of his second hour when he resumed steady incrementation from level 12 but abandoned it early in the third hour at level 20.) Only 15 puzzles worked on after the student's first rise in difficulty level from 1 to 35 were of difficulty 30 or greater; 9 of these 15 were puzzles at levels 45 to 50, clearly beyond his average competency level in series solving. The remainder of puzzles seen in the last three hours' work were primarily below difficulty level 20 and provided little opportunity for additional learning. It is likely that had the student spent more time working on puzzles between the extremes of less than 20 and greater than 45, he would have enjoyed more opportunities for solving possible series. Instead, he worked on puzzles which were trivial or too difficult.

Summary of Problem-Solving Skill Development

Problem-solving behavior changed dramatically within the student's first hour of work. Initially when the student encountered difficult series, he neglected to request additional information, tested few hypotheses, did not consider all available data, and chose to stop work too soon. By the end of the hour, more productive strategies were evident. The student learned to use "NEXT" at times when more information was needed, tested hypotheses, and persevered until there was enough information to solve series. He stopped work when his understanding was apparent. However, the majority of series worked on in the first hour were too easy for the student.

Although the student's behavior in the first hour indicated that productive problem-solving skills were available, his subsequent behavior

deteriorated in some respects. Throughout the rest of his history, "NEXT" was used effectively to gather needed additional information. However, the student stopped work too soon on numerous occasions or, conversely, persisted in entering correct responses on trivial puzzles. Difficulty levels chosen were, for the most part, either too low or too high. Therefore, the student's exposure to solvable middle-range problems was limited. It would have been in this middle range that problem-solving skills might have been most effectively applied and practiced.

Conclusions

A detailed study of one student, such as this, leaves open questions about the generality of findings. While these can only be answered by studies of more students, it is possible to supplement these findings with informal impressions from other data. In all, over 250 students have used the program for as little as 10 minutes or as long as 10 hours. One large group of students has had only one session, a controlled introduction to the program. Of those who have worked a second time, the average time at work is nearly one hour. The authors have looked at all the protocols of all the students; their impressions are qualitative.

Few students begin their work with as many skills as this subject. Most have difficulty with Skip series with differences larger than 9, with an Intersperse series of low contrast terms, and with concatenation of anything but Basic series. Further, initial skills in problem solving--particularly the use of "NEXT" to gather information, the use of "CHANGE" when the puzzle is solved, and the selection of appropriate levels of difficulty--are typically less developed than our student's skills. Most students improve in problem-solving and series-solving skills as they use the program, but only a few would surpass the performance of the student studied here.

Throughout the discussion, there have been references to inappropriate behavior by the student. One feature of the Series program is that it allows students to exhibit inappropriate and unproductive behavior in a low-threat environment. The analysis of protocols gives the researchers in this case, or possibly a teacher, an opportunity to see precisely what problem-solving skills are present and which are absent. Few, if any, environments in the school afford this opportunity.

Many of the inappropriate behaviors could be eliminated by modification to the Series program; however, most of the obvious fixes remove the possibility of inappropriate behavior, and thus the chance to observe the problem-solving skills of the student. Consider some examples:

The student did not reliably choose puzzles at difficulty levels appropriate to his skills. He spent most time working on puzzles too hard or too easy for him. One proposed modification was tested by Heller (Note 1). She modified the program to choose a new puzzle close to the current level of work. The distance of the change was related to the result on the last puzzle and the students' estimate of how difficult the puzzles were for them. No change ever exceeded three steps down or two steps up. This solution simply hides the problem. The students did progress appropriately in difficulty, but they received no instruction and no experience selecting their own level of difficulty. No evaluation of their skills in that area is possible.

A second example concerns the use of "NEXT" by the student. A general heuristic in the Series program is that after one or, at most, two errors, the student should seek more information by typing "NEXT." The student in this study grew in his ability to use "NEXT" appropriately; other students do not. A modification would force the use of "NEXT" after an incorrect guess; less drastically, the program could merely suggest the use of "NEXT." In either case, the fact that the student uses "NEXT" when directed or forced to do so gives no information about his general problem-

solving skills. It is only in the student-controlled *environment as it stands* that valid observation on this important skill is possible.

Similar modifications could be suggested to make the student change puzzles after some number of correct guesses, or after some fixed time without evident progress. These are open to the same criticism.

A second kind of modification appears more promising: The program could explicitly reinforce the behaviors believed to be productive problem solving. Most are easily detectable, based on simple counts and occurrence of events. Reinforcement could be given on various schedules when good problem solving occurs. The present program relies on the intrinsic reinforcement of "making progress" and the explicit reward of feedback on the entire puzzle. Perhaps more direct reinforcement would be more effective.

Reference Notes

1. Heller, J. Evaluation of the Series program: Effects of limiting student control on instructional effectiveness and affective outcome. Master's thesis, University of Pittsburgh, 1976.

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